Monitoring Forecast Errors with Combined CUSUM and Shewhart Control Charts[1]

Abstract

The maintenance of the accuracy (quality) of forecasts should be a constant worry for both forecasters and users of forecasts. Techniques for monitoring the underlying structure of forecasts should aim to diminish the high costs of inaccurate forecasts but also should diminish the need for unnecessary interventions in the forecasting process. This paper explores the performance of a monitoring procedure based on the use of the combined CUSUM-Shewhart Control chart. The procedure is applied to residential consumption of electrical energy for the period from January of 1994 to September of 2007 for the Brazilian State of Santa Catarina.

Keywords: Monitoring forecast errors, Combining forecasts, Control charts, CUSUM, Shewhart.

1. INTRODUCTION

One of the principal worries in forecasting is the necessity of defining the correct moment for intervening in forecasting procedures that have begun to produce intolerable inaccuracies. Interventions are justified when the size of forecasting discrepancies increase due to a change in the reality of the forecasting environment and subsequently cause an increase in cost. Interventions can take on several forms depending upon the methodologies used to generate the forecasts. When the methodologies are based on intuition and judgment, intervention in the forecasting process might take the form of an emergency meeting among top managers to investigate the inconsistencies that arise between prior knowledge and new realities. If the procedures are technical and objective, the intervention might be characterized by the estimation of new parameters, the inclusion or exclusion of variables and even drastic modifications in the underlying algorithms.

Interventions should be done sparingly to avoid the unnecessary use of resources better spent in other endeavors. In this paper, we will suggest the use of interventions that gradually increase in severity as the forecast error persists. This incremental approach to model correction should lead to cost reductions. Nevertheless, the question remains as to how to define the line between too few and too many interventions in a given period of time. We propose here the use of Shewhart and CUSUM combined control charts to help define the moment of intervention. Established monitoring practice usually observes the last recorded forecast error as indicating the need to intervene or not. However, the cumulative sum of the errors contains all the information of the historical series as to recurring discrepancies that however small in any particular period may cause significant resource loss if left ignored or unnoticed for several periods.

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The existence of forecasting discrepancies merit constant detailed monitoring, resulting in information that will contribute to the learning process of the forecaster. Instabilities in the forecast environment when monitored become knowledge of that environment, where causes and effects of troubling events can be recorded and eventually understood.

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2. METHODOLOGY

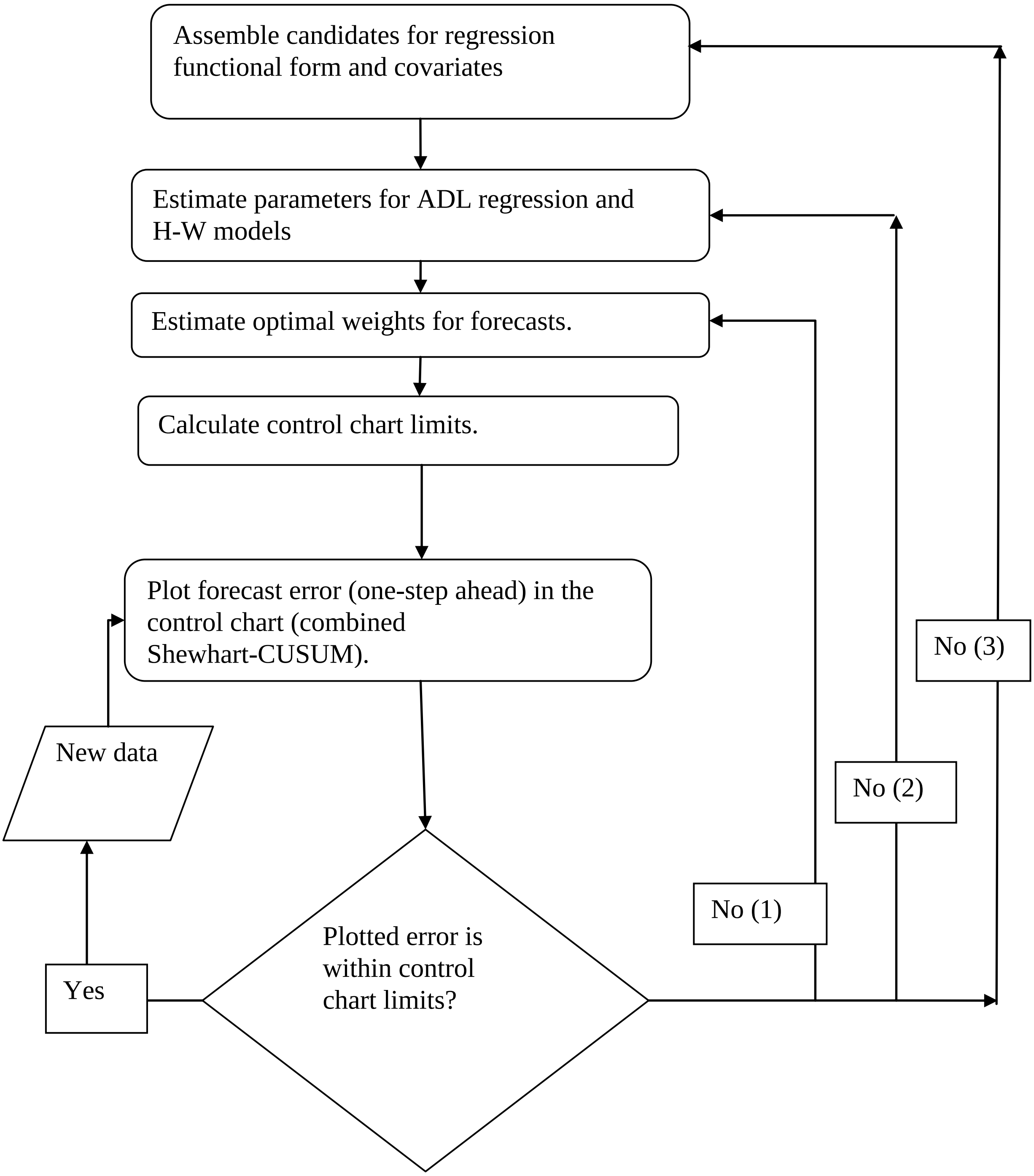
The use of control charts for monitoring forecast error has a long history in the forecast literature. The need for a rigorous treatment in monitoring forecast error was first recognized in the literature by Trigg (1964) on tracking. The Trigg statistic is calculated by dividing the individual error by its absolute value in both exponentially smoothed form and the probabilistic character of this statistic is developed. When the value is large and characterizing the tail of the distribution an alarm is sounded and the underlying forecasting model is questioned. Trigg´s equations could be cast in control chart format and will be the object of a future project by this author. The literature in CUSUM and Shewhart control charts is vast in the Engineering literature and reviewed in detail in Portuguese in Rubson (2004). The classic review of the literature in monitoring forecasts from the 1980´s is due to Gardner (1983). CUSUM in econometrics has had a profound effect on the estimation of alterations in the underlying data series and testing the null of model adequacy starting with Chow (1960). An application of CUSUM control charts to monitor forecast errors for the electrical energy sector can be found in SOUZA e SAMOHYL (2006) and other publications of Souza in the references. The initial development of the idea of statistically combining the Shewhart and CUSUM control charts is from Lucas (1982). To see the diversity of applications for CUSUM in several other fields, refer to Cowling, et al (2006) and Chan (2004).

The other technique highlighted here is the combination of forecasts, which is effectively the derivation of hybrid forecasts from several different sources. The sources could be different experts, sectors of the same company, or different methods for producing forecasts, just to mention a few. The first rigorous treatment of combining forecasts is from Granger (1974). See Armstrong (2001) for an extensive review. An interesting open question concerns the fact that even though there are complex and theoretically determined procedures for combining forecasts, in most practical applications a simple average of the individual forecasts produces the best results. Research on this topic is underway.

2.1 The monitoring process - overview

The monitoring process is a set of procedures that, with a minimum of statistical background on the part of the responsible parties, can be easily followed in the typical organizational environment. There is substantial literature on the application of forecasting procedures in the firm. See Samohyl, et al, (2008), chapter 2. Figure 1 shows the monitoring process adopted in this paper.

Figure 1 – Flowchart for the Monitoring Process



The forecasts are generated by two distinct models, the Holt-Winters exponential smoothing model[2] and an auto-regressive distributed lag (ADL) regression model. From these two distinct models, two sets of forecasts are drawn and combined using optimal weights. Even though here we use quantitative models to generate the original forecasts, other kinds of forecast procedures could be used and their forecasts optimally combined, for instance, forecasts drawn from two or more sectors of the same organization. Our choice of using the combination of forecasts in the monitoring process has two motives; one is that the literature has been sufficiently conclusive showing that combinations seem to make forecasts better, and secondly it provides a convenient step in the monitoring algorithm for easily correcting the

[2] The exponential smoothing models used here are described in Samohyl, Souza and Miranda (2008).

estimated equations before going on to more drastic overhauls. Optimal weights are calculated by ordinary least squares, restricting coefficients to be positive and to sum to one.

When standardized errors are plotted outside of the control limits, this represents an alarm that the residual is too large and therefore indicating model failure. The first step (see No (1) in figure 1) is to recalculate the optimal weights from the forecast combination and replot the newly calculated residual. The new weights may produce a smaller residual and the problem may be considered solved. If the new weights do not produce a residual sufficiently small to be contained by the control limits (see No (2)), then the underlying ADL and H-W models will be re-estimated and subsequently new optimal weights for the combined forecasts. This more profound procedure may correct the large residual and the problem of model inadequacy would be resolved. If this last procedure does not succeed in diminishing forecast error (see No (3)), then the forecasting models will have to be completely overhauled with new functional forms and variables.

In the next sections, we detail the use of control charts for monitoring countable or measurable characteristics.

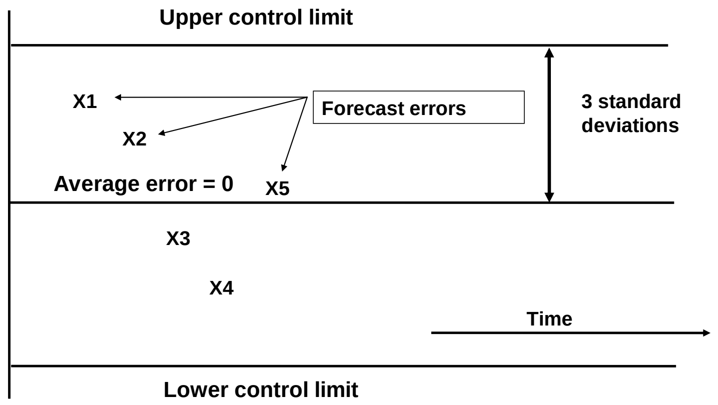
2.2 Control charts

As to the most appropriate control chart to use for monitoring forecasts, a likely candidate is the CUSUM already explored and applied by Chan (2004). Unlike the Shewhart control chart (described in the next section), the CUSUM control chart has the advantage of taking into account the history of the forecast series and is capable of detecting model failure more rapidly when forecast errors are relatively small. However, there are advantages to combining the Shewhart and CUSUM charts into a combined strategy for detecting model failure. In fact, the Shewhart chart is better than the CUSUM at detecting large modifications in forecast errors (MONTGOMERY, 2004). Therefore using both charts simultaneously with the appropriate probabilistic adjustments described below should produce quick alarms for both large model errors and repeated small ones.

2.2.1 Shewhart control chart

In figure 2, the theoretical Shewhart control chart is represented for forecast errors plotted through time. The chart itself is the result of the calculation of control limits from past data already filtered for extreme events and collection errors considered uncharacteristic of the series. This is phase 1 in the use of control charts. The chart consists of a centerline (CL) and two control limits. The center line is the average or target for the errors, desired value for forecast error is zero, and the control limits are drawn at 3 standard errors from the mean, an upper control limit (UCL) and a lower control limit (LCL). Phase 2 in the use of control charts is the reporting and plotting of forecast errors. Errors that plot outside of the limits are considered alarms that the model is failing and needs correction as described in the flowchart in figure 1. Limits at the traditional distance of 3 standard errors as depicted in figure 2 means that the probability is extremely small of an error from a non-failing model falling outside of the control limits, in fact, 0,27%, the probability of a type I error. This percentage translates into a mean false alarm rate of one in 370 forecast errors. In other words, if data are daily then on the average 370 days will occur for a false alarm to appear. Faced with a probability this small should lead the forecaster to doubt the veracity of the forecasting model if an error lies outside the limit. The control limits used in this paper are 3.25 standard errors from the centerline, which reflects a probability of type I error of 0.1%. Of course, another problem is that models may fail due to relatively small errors, and the known insensitivity of the Shewhart chart will mean a relatively long wait for a true alarm to sound. Nevertheless, the Shewhart chart is efficient for detecting model failure from large model errors close to the three standard error limit. In order to detect small deviations from target, the CUSUM chart has been proven more efficient.

Figure 2 – Shewhart Control Chart



Here the control limits were applied to one-step ahead forecast errors coming from two forecast methods: Holt-Winters and ADL regression (Phase 1) with data up to September of 2006. The one-step ahead forecasts were used in the calculation of the standard error and the charts were used for monitoring forecast error (Phase 2) starting in October of 2006. When alarms are sounded, a return to phase 1 is necessary.

2.2.2 CUSUM control chart

The CUSUM control chart has the advantage of using all information accumulated in the series, and not only data from the last period. As mentioned before, the recognition of information in the whole series leads to faster alarms for smaller modifications in the underlying reality that the model represents. The basic concept of the chart is the use of the accumulated error which should remain close to zero for an adequate model, positive and negative errors canceling out through time. If the accumulated error passes the control limit (h) then the model is considered inadequate and re-estimation is called for. Define C_t^+ as the positive sum of errors and C_t^- for the negative sum. The equation for the positive CUSUM is

$$C_t^+ = \max(0, e_t - k + C_{t-1}^+) \text{ where } e_t \text{ is the forecast error}^3$$

[3] The control chart can diminish the error of false alarms or silence in the face of model inadequacy (type II error, non detection of model failure) by fixing a band around the forecast error where small errors are considered null, called a reference value k, however, not used here.

There is a similar equation for the negative sum C_t^-. Here we use a value of 4.5 standard errors calculated from the one-step ahead forecasts for the value of h. In the Brazilian electrical energy sector, the penalties applied to companies that produce repeated inaccurate forecasts however small are substantial, and the value for h (= 4.5) chosen for this project would be considered very low for a majority of applications. This low value for h might incur heavy resource use involved in the frequent recalculation of the forecasting models, however, considering the high cost of forecast error the energy company chose to be cautious instead of remorseful.

2.2.3 Combining two control charts in one

As described above, to avoid false alarms (type I error, rejecting the true null) control limits are located relatively far from the central tendency of the data (CL in the control chart) where there is only a very small probability of a stable process producing a point outside of the limits. This false alarm rate uses the symbol α. When two different control charts monitor the same variable, each chart has its own α, for instance α_S and α_C for the Shewhart and CUSUM charts. Consider α_S and α_C both equal to 1%, then the false alarm rate for the two combined charts would be

$$\alpha = \alpha_S + \alpha_C - \alpha_S{*}\alpha_C = 1\% + 1\% - 1\%{*}1\% = 1.99\%$$

Note that when the two charts have a 1% false alarm rate the overall false alarm rate for the two charts used together is practically 2%. If three charts were used then the false alarm rate would be about three per cent, and so on. In light of this fact, control limits should be recalculated (widened) to return the false alarm rate back to its original value. For this case, each individual α_C and α_S would have to be about 0.5% to make the combined α 1%. For this project, the false alarm rates were α_S = 0.1% (3.25 standard errors), and α_C = 6.2% (4.5 standard errors). Therefore, the overall false alarm rate is about 6.3%. The probability of getting two false alarms together is 6.3%*6.3% = 0.4%. This probability is very low and demands that the forecasting models be re-estimated as suggested in the monitoring algorithm.

2.2.4 Data and case study

The monitoring algorithm from figure 1 is applied to forecast errors for the residential consumption of electrical energy in the State of Santa Catarina. The forecast period is from October of 2006 to September of 2007. The forecasts are the result of the combination of the two aforementioned methods, ADL regression and Holt-Winters. As will be seen below, two distinct months are problematic and two interventions will be required to reset the parameters of the forecast models.

3. MONITORING PROCEDURE APPLIED – ELECTRICAL ENERGY COMPANY OF SANTA CATARINA.

As can be seen from the flow chart in figure 1, the methodology reported here consists of basically three steps: (a) the construction of the forecasts, (b) the combination of the forecasts, e finally (c) the monitoring of the forecasts and the veracity of the underlying model. The results are presented below.

3.1 Constructing forecasts

In the regression model, to explain residential consumption of electrical energy (CR) measured in MWh, several explanatory variables were used.

ICMS-Deflac – Deflated state value added tax
SM – national deflated minimum salary
IGP-M – General price index
TC – real exchange rate
TJ – interbank interest rate
DUI – business days in the month.

In equation 1.1, table 1 and in an annex to this article the results for the ADL regression are shown. The dependent variable D2CR is the second difference of monthly residential consumption. The calculations were done in the econometric software PCGIVE and PCGETS.

$$\begin{aligned}
D2CR_t &= 0.2602\ D2CR_{t-1} - 0.8044\ D2CR_{t-2} + 0.2321\ D2CR_{t-3} - 0.8384\ D2CR_{t-4} \\
&- 0.6542\ D2CR_{t-6} + 0.3083\ D2CR_{t-7} - 0.5495\ D2CR_{t-8} - 0.3672\ D2CR_{t-10} \\
&+ 13.076\ ICMS\text{-}Deflac_t + 16.756\ ICMS\text{-}Deflac_{t-1} - 16.355\ ICMS\text{-}Deflac_{t-5} \\
&- 171.501\ SM_{t-1} + 225.598\ SM_{t-7} + 4743.17\ IGPM_{t-1} + 2409.73\ IGP\text{-}M_{t-5} \\
&+ 894.991\ IGP\text{-}M_{t-8} - 536.428\ IGP\text{-}M_{t-11} - 712.213\ TC_{t-2} + 723.645\ TC_{t-3} \\
&- 721.43\ TC_{t-4} - 816.9\ TC_{t-5} - 609.714\ TC_{t-6} + 205.484\ TC_{t-11} + 7099.34\ TJ_{t-1} \\
&- 3268.88\ TJ_{t-8} - 3803.79\ DUI_{t-4} + 4155.18\ DUI_{t-6} - 3747.7\ DUI_{t-8} - 2597.06\ DUI_{t-10}
\end{aligned} \tag{1.1}$$

The results for the Holt-Winters model and some further results for the regression (1.1) are in table 1. The regression results show that the regression residuals are very well-behaved. Note that the Holt-Winters parameters demonstrate that the data are highly seasonal as would be expected.

Table 1 – Characteristics of the two methods used for calculating forecast models for the period Jan/94 a Sep/06.

ADL REGRESSION MODEL (equation 1.1)								
Sigma	MAPE	U - Theil	R^2_{Aj}	AR test*	ARCH test*	Normality test*	Hetero test*	RESET test*
8319.83	2.69%	0.5180	0.81	0.9125	0.6898	0.9362	0.3246	0.4158
HOLT-WINTERS								
Sigma	MAPE	U - Theil	Alfa	Beta	Gama	-	-	-
8739,06	2.65%	0.5405	0.2787	0.0629	0	-	-	-

* p-value.

Both methods produce accuracy measures (MAPE, sigma and U-Theil) that are very similar.[4]

[4] The Holt-Winters procedure is known for its low cost of application. In the event of choosing one of the methods as "the best", we would be inclined to choose Holt-Winters.

3.2 Combining forecasts

Forecasts were generated from the two methods and were combined using the criterion of minimum least square error. The results in table 2 show that the Holt-Winters method, assuming a large weight of almost 62%, is preferred to the regression model whose weight is only 38%. It is also interesting to note the difference in the accuracy measures sigma, MAPE and U-Theil that are all "better" for the combined forecasts.

Table 2 – Optimal weights for the two forecasting methods, data from Jan.1994 to Sep.2006.

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Sigma	MAPE	U - Theil	Observations	Regression model ADL	Holt-Winters
7535.79	2.36%	0.4742	139	38.12%	61.88%

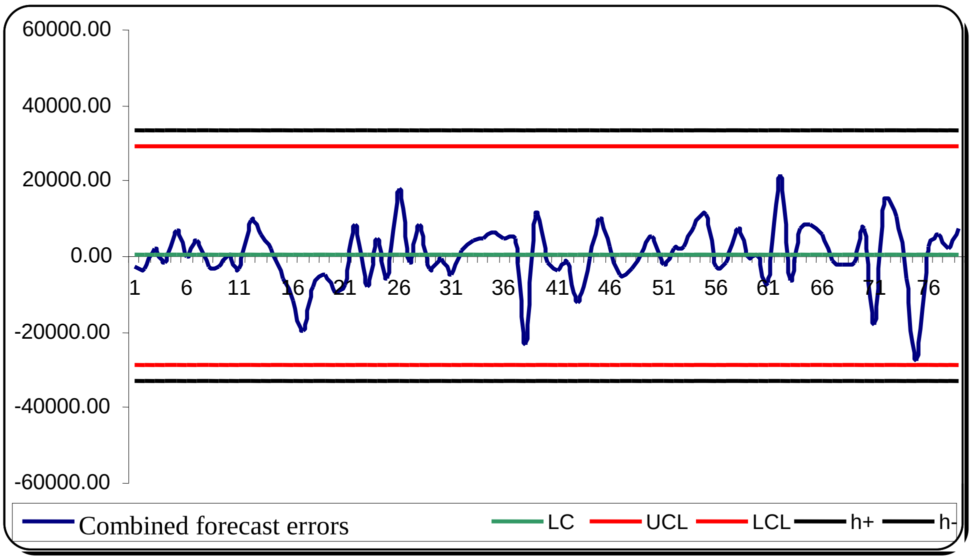
3.3 Monitoring the combined forecasts

The first month of monitoring is October of 2006. See figure 4. The standard error for the one-step ahead forecasts is 8306 MWh and the mean is 276.96 MWh. These values when applied to the parameters of the Shewhart - CUSUM control charts (3.25 e 4.5σ) generate the following limits:

Upper control limit for Shewhart (UCL) (276.96 + 3.25*8306 =) 27271 MWh,
Lower control limit (LCL) (276.96 - 3.25*8306 =) -26717 MWh,
h_+ (CUSUM upper limit) (4.5*8306) = 37377 MWh e
h_- (CUSUM lower limit) = -37377 MWh.

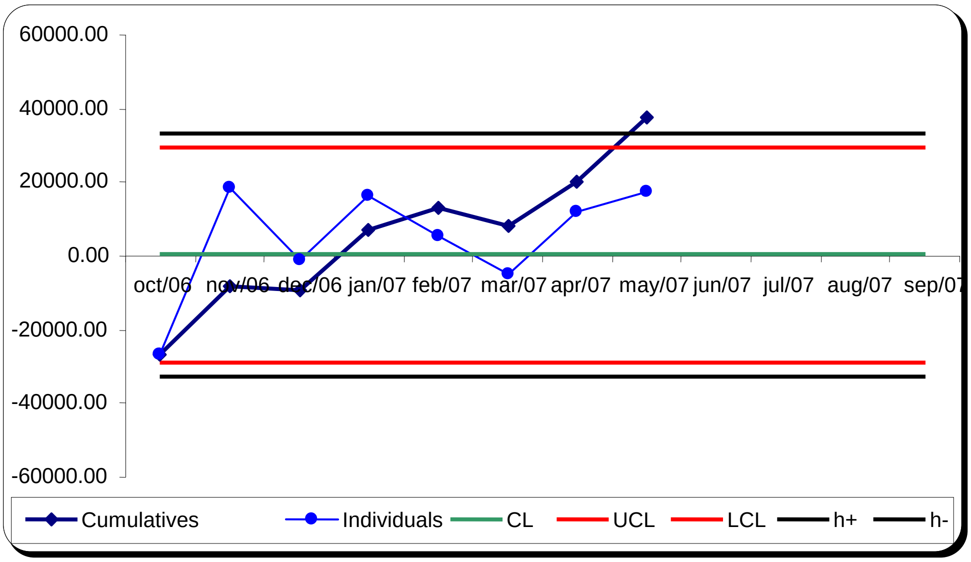
Figure 3 corresponds to the phase 1 of constructing the control chart from data up to September of 2006. The limits shown here are used for monitoring the first months (beginning in October of 2006) of the forecast errors.

Figure 3 – Combined control chart CUSUM-Shewhart with sigma = 3.25 e h = 4.5σ (sample period up to Sep. 2006).



Monitoring with the limits of figure 3 continued until May of 2007 when an alarm occurs. During 8 months, the month by month forecast calculations produced small errors and consequently there was no need for intervention in the forecasting process. In figure 4, the control chart shows relative stability in the forecasts until the month of May when an alarm is sounded by the CUSUM part of the chart. At no time did the individual error overcome the Shewhart limit demonstrating the sensitivity of the CUSUM chart to detect model failure due to small changes in the underlying environment.

Figure 4 – Monitoring forecast errors in a combined Shewhart and CUSUM control chart.

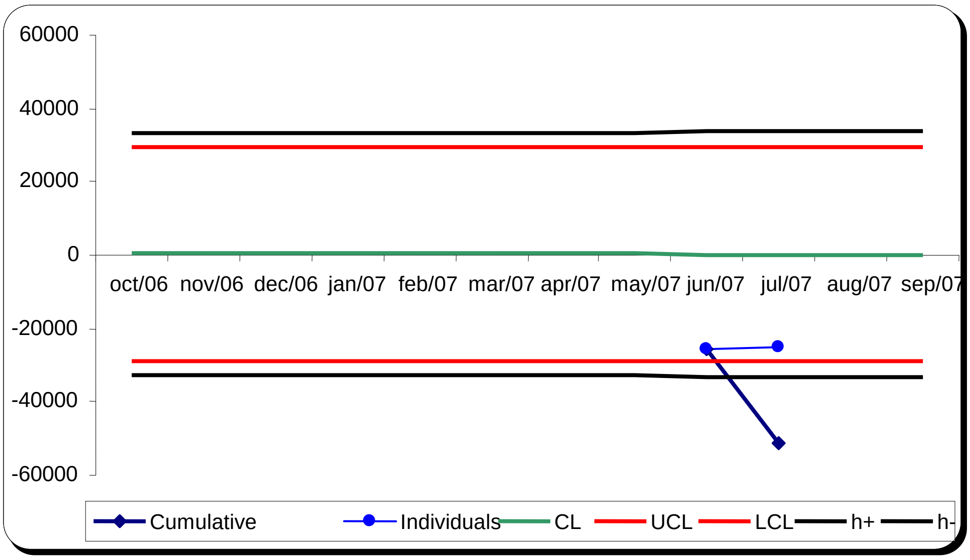


According to the monitoring algorithm set forth in figure 1, in light of the alarm present in figure 4, the optimal weights for combining forecasts from the two models need to be recalculated. The updating of the parameters for combining forecasts after the alarm uses data up to May of 2006, the month of the alarm. We do not admit yet the alteration of the individual model parameters which will occur only if a second point appears outside of the control limits.

After the realization of the intervention, the new optimal weights were altered to 35.94% e 64.06%, MAPE increased to 2.46% and U - Theil increased to 0.4881. The new standard error increased to 8390.28 MWh and the new average changed to 267.94 MWh. See table 3. In general the new calculations show a worsening of the series from the viewpoint of prediction accuracy, but this is to be expected considering that the data from May have been shown to be more deviant from the rest. The new upper control limit is (267.94 + 3.25*8390 =) 27534,5 the lower control limit is -27000 MWh, h_+ is 37755 MWh and h_- is -37755 MWh.

Figure 5 shows the results for the new updated control chart. In June there were no problems, the plots are inside all control limits. However, in July of 2007 once again the CUSUM part of the control chart emits an alarm, this time lower than the negative h_-.

Figure 5 – Monitoring forecast errors in a combined Shewhart and CUSUM control chart after the first intervention in May of 2007.



With the second intervention, the optimal weights change again, the regression weight falling to 34.55% and the Holt-Winters weight rising to 65.45%. In table 3 and 4, the results from the second intervention are grouped with the first intervention, and the original calculations before any interventions. We notice that in light of the increase in the standard error after each intervention, the control limits tend to increase their distance from the centerline in terms of MWh´s.

Table 3– Re-estimation of optimal combining weights.

ORIGINAL ESTIMATION (sample from Jan.1994 to Sep.2006)									
Weight Regression	Weight HW	MAPE	U de Theil	Standard error	CL	UCL	LCL	h+	h-
38.12%	61.88%	2.36%	0.4742	8305.82	276.96	27271	-26717	37377	-37377
FIRST INTERVENTION (sample estimates to May, 2007)									
35.94%	64.06%	2.46%	0.4881	8390.28	267.94	27534	-27000	37755	-37755
SECOND INTERVENTION (sample estimates to Jul.2007)									
34.55%	65.45%	2.55%	0.5051	8445.06	262.21	27708	-27184	38002	-38002

In figure 6, monitoring after the second intervention does not produce an alarm up to September of 2007 when the project ended, meaning that the parameters of the models represent reasonably well the underlying environment. In table 5, all of the control chart results are recorded.

Figure 6 – Monitoring forecast errors in a combined Shewhart and CUSUM control chart after the second intervention in July of 2007.

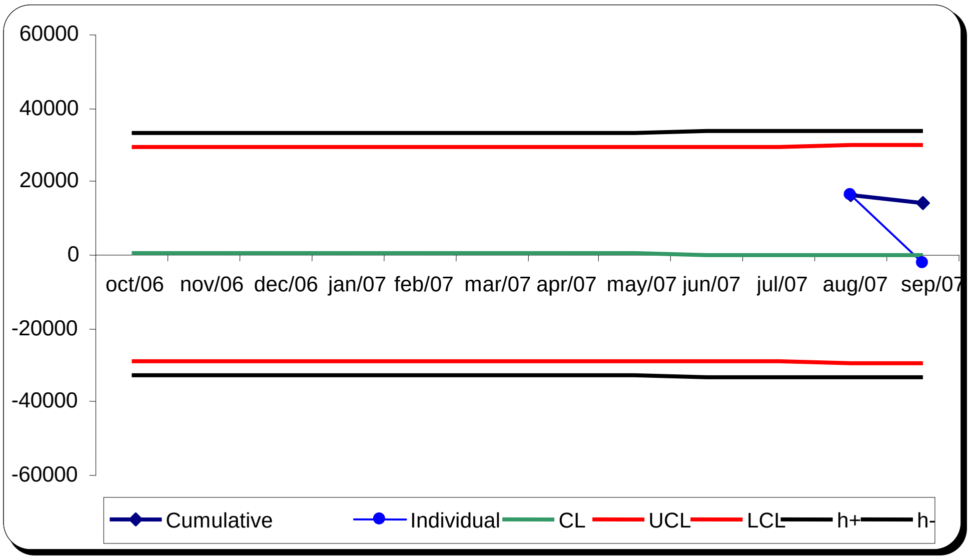


Table 4 – Historical process for monitoring forecast errors and points of intervention

MONITORING THE FIRST MONTHS (sample estimates from Jan.1994 to Set.2006)								
	Error Individual	Error Cumulative	CL	LCL	UCL	h+	h-	APE
Oct/06	-26778.02	-26778.02	276.96	-26717	27271	37377	-37377	10.04%
Nov/06	18442.11	-8335.91	276.96	-26717	27271	37377	-37377	6.20%
Dec/06	-846.31	-9182.22	276.96	-26717	27271	37377	-37377	0.30%
Jan/07	16349.62	7167.41	276.96	-26717	27271	37377	-37377	4.89%
Feb/07	5719.38	12886.78	276.96	-26717	27271	37377	-37377	1.70%
Mar/07	-4800.15	8086.63	276.96	-26717	27271	37377	-37377	1.47%
Apr/07	12249.01	20335.64	276.96	-26717	27271	37377	-37377	3.72%
May/07	17468.18	37803.82*	276.96	-26717	27271	37377	-37377	5.45%
FIRST INTERVENTION (sample estimates to May, 2007)								
Jun/07	-25870.37	-25870.37	267.94	**-27000**	**27534**	**37755**	**-37755**	9.39%
Jul/07	-25208.05	-51078.42*	267.94	**-27000**	**27534**	**37755**	**-37755**	9.30%
SECOND INTERVENTION (sample estimates to Jul.2007)								
Ago/07	16542.82	16542.82	262.21	**-27184**	**27708**	**38002**	**-38002**	5.40%
Sep/07	-2254.47	14288.36	262.21	**-27184**	**27708**	**38002**	**-38002**	0.74%

CL = centerline; LCL = lower control limit; UCL = upper control limit; h+ = upper limit CUSUM; h- = lower limit CUSUM; APE = absolute percentage error. * signifies a value that is beyond the control limit.

4. CONCLUSIONS

The project ended in September of 2007 and the research group has not had further official contact with the Electrical Energy Company. Nevertheless, the successfulness of the project is a well-known fact, and since that date, it has not been necessary to re-estimate the optimal weights nor the parameters of the forecast models. Mission accomplished.

5. SPECIAL THANKS

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7. ANNEX

Statistical results for equation 1.1

Variável	valor t	p-valor
$D2CR_{t-1}$	4.56	0
$D2CR_{t-2}$	-12.7	0
$D2CR_{t-3}$	3.56	0.001
$D2CR_{t-4}$	-13.5	0
$D2CR_{t-6}$	-9.3	0
$D2CR_{t-7}$	4.42	0
$D2CR_{t-8}$	-7.49	0
$D2CR_{t-10}$	-5.43	0
ICMS-$Deflac_{t}$	2.33	0.022
ICMS-$Deflac_{t-1}$	2.83	0.006
ICMS-$Deflac_{t-5}$	-2.98	0.004
SM_{t-1}	-2.82	0.006
SM_{t-7}	3.47	0.001
IGP-M_{t-1}	3.76	0
IGP-M_{t-5}	2.05	0.043
IGP-M_{t-8}	2.87	0.005
IGP-M_{t-11}	-3.04	0.003
TC_{t-2}	-3.47	0.001
TC_{t-3}	2.65	0.009
TC_{t-4}	-2.56	0.012
TC_{t-5}	2.87	0.005
TC_{t-6}	-3.21	0.002
TC_{t-11}	2.65	0.009
TJ_{t-1}	3.82	0
TJ_{t-8}	-2.21	0.029
DUI_{t-4}	-5.36	0
DUI_{t-6}	5.53	0
DUI_{t-8}	-4.16	0
DUI_{t-10}	3.66	0